The Vitruvian Man, Nature, and Breakthrough Applications Brought to Life

Using The Fibonacci Numbers and The Golden Ratio

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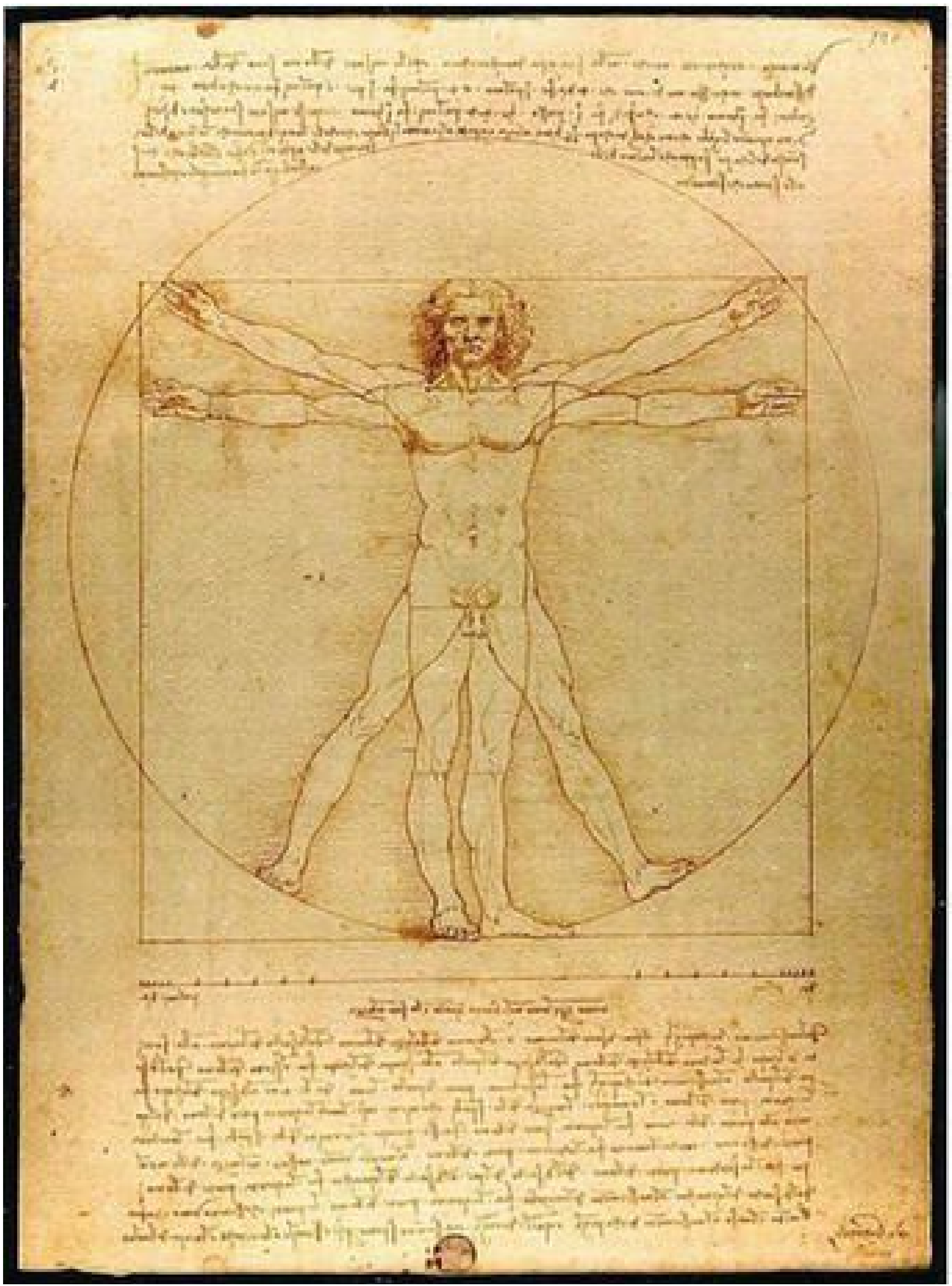


ABSTRACT

Fibonacci numbers and the Golden Ratio are two mathematical constructs found throughout nature and art and are present all around us, including in our own bodies and the bodies of structures of all living things no matter their biological kingdom. The Fibonacci sequence is a series of numbers in which each number is the sum of the two numbers preceding it. Examples of the Fibonacci numbers can be found throughout the human body's structures, such as one head and two ears, one nose and two nostrils, one finger each with two knuckles and three bone segments, and much more. The mathematical ratio expressed as the irrational number 1.618∞, and which is deeply intertwined with the Fibonacci sequence, is known as the Golden Ratio, or φ (Phi). Its ratio measurement is so fundamental and universal that even living things find themselves based upon it. For example, the ratio is found between body parts, such as the measurements between the wrist to the elbow as well as between the shoulder line and the top of the head. This paper will delve into the background of the Fibonacci sequence and the Golden Ratio as well as its current and future applications. The Fibonacci number sequence and the Golden Ratio are fundamentally helpful within a wide variety of biological, medical and engineering applications today, and will continue to form the basis for scientific and technological advances well into the future.

INTRODUCTION

The Fibonacci numbers were discovered by Leonardo Pisano, also known by his nickname, Fibonacci. In 1202 AD Fibonacci wrote the book, Liber Abaci that described this elegant numerical sequence which came to bear his name, and that is also the foundation of the Golden Ratio (φ). This sequence was already known and used as early as the sixth century AD by Indian mathematicians but Fibonacci was the first to introduce this concept to the "Western" world. The Fibonacci sequence is quite simple, actually. It is a series of numbers that follows the rule that each number is the sum of the two numbers preceding it. The Fibonacci sequence goes on infinitely as we can see as we begin listing it below:

1,1,2,3,5,8,13,21,34,55,89,144,233,377, ….. ∞

1+1=2

1+2=3

2+3=5

3+5=8

5+8=13

8+13=21

…. ∞

The Fibonacci Rule is a mathematical way to find certain numbers in the Fibonacci sequence. The rule is a basic equation that explains the rule of the sequence. It can be simplified in these terms: $x_n = x_{n-1} + x_{n-2}$ The Fibonacci Numbers are defined by the recurrence relation defined by the equations $x_n = x_{n-1} + x_{n-2}$ for all $X \geq 3$ where $X_1 = 1$; $X_2 = 1$; $X_3 = 2$ and where X_n represents the nth Fibonacci number in the sequence (n is called an index). To use this formula, first plug in the number of the sequence you are trying to find for "n" and solve.

For example, if we are trying to find the 10th term in the sequence, we can simply use the sequence's known numbers and applying basic arithmetic:

$x_n = x_{n-1} + x_{n-2}$

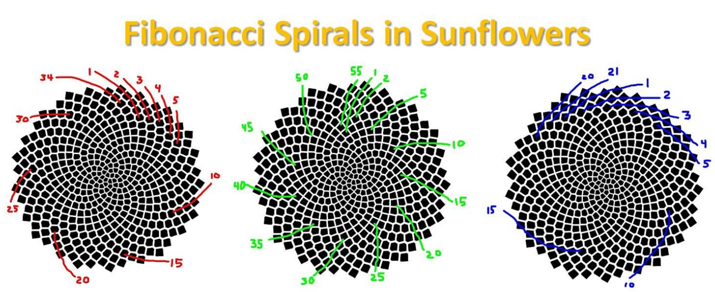
$x_{10} = x_{10-1} + x_{10-2}$

$= x_9 + x_8$

$= 34 + 21$

$= 55$

The Fibonacci sequence can also be found in many aspects of nature, such as petal or leaf patterns. To expand, on the head of a sunflower, the seeds are packed in a certain way so that the ordering of the seeds on the flower's head follow the pattern of the Fibonacci sequence and relate to each other according to the Golden Ratio (which we will explore in further detail below). The resulting spiral pattern prevents the seeds of the sunflower from overlapping or overcrowding yet



maximizes the number of seeds allowed. Through millennia of natural selection, the sunflower incorporated the Fibonacci sequence and Golden Ratio in its very structure to maximize the chances for the continuation of the species. Nature isn't trying to use the Fibonacci numbers: they are appearing as a by-product of a deeper physical process. That is why the spirals are imperfect. The plant is responding not to a mathematical rule but takes into account any natural constraints while it attempts unknowingly to "follow" the Fibonacci sequence and Golden Ratio to achieve the optimal configuration of its seed pattern,

{Picture from https://momath.org/home/fibonacci-numbers-of-sunflower-seed-spirals}

The Golden Ratio, which was first discovered by Greek Mathematicians and Philosophers, is a mathematical proportion and ratio where the length to the width of a rectangle is 1:1.618033988749∞…. This number is a never ending, irrational and very special number, and is also known as Phi or φ. The ratio is used in many forms of art, such as proportions for drawing a human eye, as well as Da Vinci's Vitruvian Man, an artwork resembling the "ideal" or "perfect" human form using the Golden Ratio proportions and Fibonacci Numbers. Upon study by archaeologists and mathematicians, the Golden Ratio was also found in Khufu's Pyramids as well as the Pyramids of Giza. However, the ratio is not only found in ancient times, it is still continually used today in animations, anatomy understanding, biometrics, plastic surgery, simulation software and much more.

The Golden Ratio is believed to make the most pleasing and beautiful shape. It appears many times in geometry, art, architecture and other areas and is important in a wide variety of ways. In order to understand its applications, we need to understand how and why it works. The

Golden Ratio, also known as Phi or the Divine Proportion, has both aesthetically pleasing qualities in nature as well as interesting and special properties.

We can find the Golden Ratio when we divide a line into two parts so that the long part divided by the short part is equal to the whole length divided by the long part. This can be shown by the formula $a/b = (a + b)/a$.

We saw above that the Golden Ratio has this property:

$$a/b = (a + b)/a$$

We can split the right side of the fraction like this:

$$a/b = a/a + b/a$$

To explain the meaning of this equation, a/b is defined as the Golden Ratio (φ), $a/a=1$ and since a/b is φ, b/a is the inverse of that making it equal to $1/\varphi$, which simplifies the equation to $\varphi = 1 + 1/\varphi$. So the Golden Ratio has a recursive relationship, meaning it can be defined in terms of itself. To visualize this, the equation can be simplified using a 1x1 square. Next find the square's midpoint and connect that midpoint to the opposite corner of the square. After this, use this line as a radius to draw an arc extending out of the original square and connect the points to form a rectangle. This special rectangle is also known as the Golden Rectangle and is one of the many ways the Golden Ratio can be applied to modern day usage.

In order to calculate φ, you can use the formula above, $\varphi = 1 + 1/\varphi$. First, plug in a random value to start, then do this calculation again and again until you reach φ.

A) divide 1 by your value (1/value)

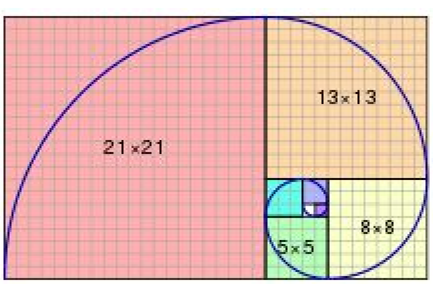
B) add 1

C) now use that value and start at step A again

Value	**1/Value**	**1/Value + 1**
4	¼ = .25	.25 + 1 = 1.25
1.25	1/1.25 = .8	1.8
1.8	0.555555555556	1.555555555556
1.555555555556	0.642857142855	1.642857142855
1.642857142855	0.608695652175	1.608695652175
1.608695652175	0.621621621621	1.621621621621

As we keep following the formula, each successive "run" of the formula results in an answer that is closer and closer to φ (or 1.618...). You will find that no matter what number you start with (in the example above, I started with the number 4) you will always wind up at φ. However, using this method is not very efficient, especially when the numbers become larger and larger.

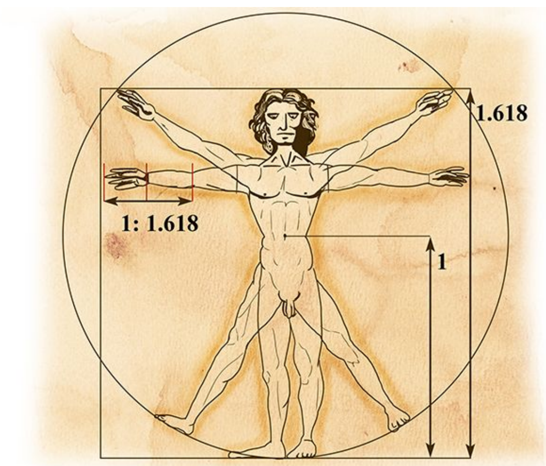
In addition to trying to find the value, you can also use the formula and rules in order to draw the Golden Ratio Spiral. In order to do this, you start by drawing a 1x1 square, then another 1x1 square adjacent to and above it. Using the two side lengths draw a 2x2 square adjacent to



and next to the two 1x1 squares. Continue this pattern using squares of dimensions such that each new square fits adjacent to the two preceding it (refer to the figure - you will discover that the successive dimensions of these squares are none other than the Fibonacci Numbers!).

Picture from https://commons.wikimedia.org/wiki/File:GoldenSpiralLogarithmic_color_in.gif

The Vitruvian Man was a drawing illustrating the concepts described by the Roman Architect Vitruvius in the 1st Century BC. The drawing was created in 1500 AD by Leonardo Da Vinci. However, many do not know that this magnificent piece of artwork beholds many scientific and mathematical principles including the Golden Ratio. Da Vinci was not only an artist, but a dedicated scientist and engineer. The Vitruvian Man incorporates all of these qualities through the application of mathematics. Throughout the piece, the Golden Ratio is present including:



the navel to the top of the hairline

the width of the chest to the width of the collarbones

the distance from the Da Vinci’s guide line drawn at the elbow to the guideline at the fingertips

APPLICATIONS TO REAL WORLD PROBLEMS

The Golden Ratio and Fibonacci numbers are used in a wide variety of ways both in man made endeavors as well as in the natural order of the Universe. As discussed above, the Golden Ratio can be found in a wide range of living things, as well as in architecture and artwork, from Da Vinci's Paintings to The Great Pyramids of Giza and Khufu. Luca Pacioli, a modern day Da Vinci, displays his thoughts on the subject by saying, "without mathematics there is no art." In my opinion, proportions and angles are necessary in order to sculpt or paint a human body or face, as well as being an essential part of design, visual aesthetics, and beauty. Throughout history, the Golden Ratio was used for many well known artworks and architectural designs and structures, and it is still used quite frequently in our society and scientific endeavors today, as well as being found throughout the human body and within the engineering and even "growth" of body parts.

The Golden Ratio is found repeatedly in the human face and is used as guidelines for many plastic surgeons. One example is found from the bottom of the chin to the hairline in length to the edge of the eyebrow. This proportion ends up following φ and is displayed as such: 1:1.618 (or 1: φ). Another example is based on the building blocks of φ as φ can be broken down into the equation: $5^{.5} \cdot .5 + .5 = \varphi$. This example revolves around the fact that the human body is based on φ and 5.

5 appendages to the torso, in the arms, leg and head.

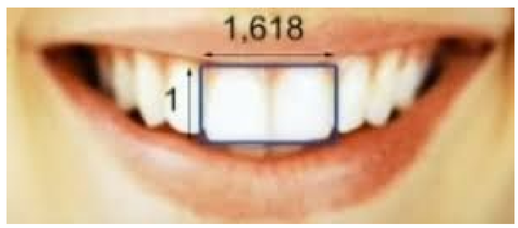
5 appendages on each of these, in the fingers and toes

5 openings on the face.

5 sense organs for sight, sound, touch, taste and smell.

It is to be noted that the connections between properties grow as the number 5 is the fifth number in the Fibonacci sequence as well as being directly tied to the Golden ratio as well as our own human anatomy.

In addition, the ratio and Fibonacci sequence can also be found in a large variety of dental procedures and tooth shaping. For example, the total width of the two front teeth in the upper jaw over their height should produce the Golden Ratio. Furthermore, the width of the first tooth from the center to the second tooth also yields the Golden ratio. These proportions are ideal in tooth structure, so may be considered when molding replacement teeth or reshaping those already permanent.



Some other golden ratio measurements include:

Length of face / width of face

Distance between lips and where the eyebrows meet/length of nose

Length of face / distance between tip of jaw and where eyebrows meet

Length of mouth / width of nose

Width of nose / distance between nostrils

Distance between pupils / distance between eyebrows

These measurements are considered “perfect” features and would be displayed in the “perfect” face. Although not many people are born with all of these perfect ratios (because, unlike most other animals and plants, humans tend to choose their mates for many reasons beyond purely physical traits) , they are extremely eye pleasing and sought after by many, especially in plastic surgery. Many plastic surgeons follow these measurements to give their patients the “ideal face or body”.

Interestingly enough, not only are our outside features composed of the Fibonacci sequence and Golden Ratio, but the inner workings of our own body are made up of these two properties. One feature is found in the system of bronchi which makes up our lungs. To expand, the main windpipe divides into two main bronchi, one short (right bronchi) and one long (left o90-bronchi). Through extensive research, it was found that the proportion of the short bronchus to the long was always 1:φ. Even more fascinating, the DNA molecule, the program for all life is based on φ. The molecule measures 34 Å (Angstroms*) to 21 Å (Angstroms), which are numbers in the Fibonacci sequence, and their ratio closely approximates φ at 1:1.6190476. These remarkable connections make the Fibonacci sequence and Golden Ratio fascinating topics of study.

*Angstroms are a unit of measurement that is one ten-billionth of a meter, approximately the size of an atom.

WHAT IF & CONCLUSION

Today we currently use the Golden Ratio and Fibonacci Numbers to create the "ideal look" for the face or body as well as for creating the right size for prosthetic limbs. I would like to identify more applications and study applications of the Golden Ratio and Fibonacci Numbers as I described the lungs above. Using those measurements, the Fibonaccis and the Golden Ratio could potentially help us build synthetic printed lungs and other organs for transplant. A study in the Hindawi journal on the weight ratio of organs concluded that the heart, liver, left kidney, brain, and left lung were positively correlated to body weight while only the brain and left lung were correlated to height in men. In females, the heart, liver, right kidney, brain and right lung were positively correlated to the weight of the body while the right kidney only correlated to the height of the body. I believe that these facts and observed ratios under further observations and study will no doubt correlate to the Golden Ratio and Fibonacci sequences, and will help scientists come up with a systematic mathematical formula to help them print exact replicas and exact sizes of organs needed for successful transplants in the future. The possibilities seem endless and could equally extend to the atomic and subatomic scales, with applications in materials, healthcare, computing, energy and more.

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